

# A new generation of Lepton Flavour Universality observables from $B \rightarrow K^* \mu^+ \mu^-$

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The recent years of activity at the LHC have brought to light several anomalies in exclusive semileptonic  $b \rightarrow s \ell^+ \ell^-$  decays that are shortly reviewed here. Our latest global model independent analysis is pointing to sizable New Physics contributions to different Wilson coefficients (especially  $C_9$ ). An accurate analysis of hadronic uncertainties shows that the few attempts in the literature to explain the anomalies using factorizable or non-factorizable power corrections fail. Finally, we propose a new set of observables able to test for lepton-flavour universality violation and free of hadronic uncertainties in the Standard Model. The properties of these observables are analysed within the Standard Model as well as within several New Physics benchmark scenarios.

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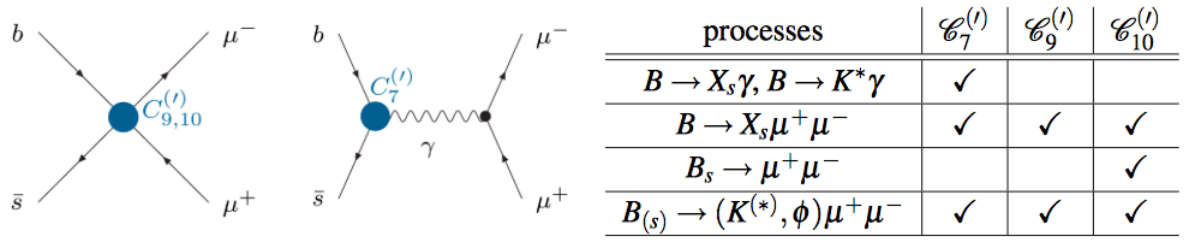
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## 1. Introduction

After the Higgs search era at the LHC, flavour physics has become one of the most promising windows for studying possible Beyond the Standard Model (BSM) effects. In particular, processes driven by the flavour-changing neutral current (FCNC) transition  $b \rightarrow s \ell^+ \ell^-$  have been providing hints of several deviations from the Standard Model (SM) in various observables. In 2015, the LHCb analysis [1] of the  $3 \text{ fb}^{-1}$  data on  $B \rightarrow K^* \mu^+ \mu^-$  confirmed a  $\sim 3\sigma$  anomaly in two large  $K^*$ -recoil bins of the angular observable  $P'_5$  [2, 3] that was already present in the 2013 results with  $1 \text{ fb}^{-1}$  [4]. The same experiment measured the observable  $R_K = \mathcal{B}(B \rightarrow K \mu^+ \mu^-) / \mathcal{B}(B \rightarrow K e^+ e^-)$  [5] in the dilepton mass range from 1 to 6  $\text{GeV}^2$  and found a  $2.6\sigma$  tension with its SM value, predicted to be equal one (to a one per cent accuracy). Further deviations were observed by LHCb in two of the large-recoil bins of the branching ratio of  $B_s \rightarrow \phi \mu^+ \mu^-$  [6] but also in the low-recoil bins of  $B^+ \rightarrow K^{*+} \mu \mu$  and  $B_s \rightarrow \phi \mu \mu$  with a  $2.5\sigma$  and  $2.2\sigma$  deviation respectively. Indeed a systematic trend of experimental data preferring values below SM expectations in semileptonic  $b \rightarrow s \ell \ell$  decays is observed<sup>1</sup>. And finally, last year, the Belle experiment performed an independent measurement of  $P'_5$  [7] confirming the deviation w.r.t. SM observed by LHCb.



**Figure 1:** Effective couplings  $C_{7,9,10}^{(\prime)}$  contributing to  $b \rightarrow s \ell^+ \ell^-$  transitions and sensitivity of the various radiative and (semi-)leptonic  $B_{(s)}$  decays modes to them.

The relevance of the above mentioned tensions is that all of them are sensitive to the same effective couplings, providing a perfect testing ground of the existence of coherent patterns among the observed anomalies. The effective couplings are the Wilson coefficients  $C_{7,9,10}^{(\prime)}$  of the four-fermion operators in the effective Hamiltonian approach (see Fig. 1):

$$\mathcal{O}_7^{(\prime)} = \frac{\alpha}{4\pi} m_b [\bar{s} \sigma_{\mu\nu} P_{R(L)} b] F^{\mu\nu}, \mathcal{O}_9^{(\prime)} = \frac{\alpha}{4\pi} [\bar{s} \gamma^\mu P_{L(R)} b] [\bar{\ell} \gamma_\mu \ell], \mathcal{O}_{10}^{(\prime)} = \frac{\alpha}{4\pi} [\bar{s} \gamma^\mu P_{L(R)} b] [\bar{\ell} \gamma_\mu \gamma_5 \ell]$$

whose values in the SM are:  $C_7^{\text{SM}}(\mu_b) = -0.29$ ,  $C_9^{\text{SM}}(\mu_b) = 4.07$ ,  $C_{10}^{\text{SM}}(\mu_b) = -4.31$ , and where  $P_{L,R} = (1 \mp \gamma_5)/2$ ,  $m_b$  stands for the mass of the  $b$  quark and  $\mu_b = 4.8 \text{ GeV}$  denotes the energy scale. The right-handed (primed) Wilson coefficients are not given since they either vanish or can be neglected in the SM.

The structure of this proceeding is the following: in Sec.2 we report the features and most important results of the analysis in Ref. [8]. The origin of the hadronic uncertainties and how they

<sup>1</sup>There is only one exception to this global trend: the low-recoil region of  $\Lambda_b \rightarrow \Lambda \mu^+ \mu^-$  (interestingly the large-recoil region exhibit the same deficit as the rest of decays). The error size in the low-recoil region is very large to draw any definite conclusion and it is mainly driven by the normalization channel  $\Lambda_b \rightarrow \Lambda J/\psi$ . More data, specially regarding the normalization, will help to clarify the situation.

Coefficient	Best fit	$1\sigma$	$3\sigma$	Pull <sub>SM</sub>	p-value (%)
$C_9^{\text{NP}}$	-1.05	$[-1.25, -0.85]$	$[-1.62, -0.40]$	<b>4.7</b>	61.0
$C_9^{\text{NP}} = -C_{10}^{\text{NP}}$	-0.59	$[-0.74, -0.44]$	$[-1.06, -0.17]$	<b>4.3</b>	51.0
$C_9^{\text{NP}} = -C_{9'}^{\text{NP}}$	-1.00	$[-1.20, -0.78]$	$[-1.55, -0.32]$	<b>4.4</b>	54.0
$C_9^{\text{NP}} = -C_{10}^{\text{NP}} = -C_{9'}^{\text{NP}} = -C_{10'}^{\text{NP}}$	-0.61	$[-0.45, -0.45]$	$[-1.17, -0.17]$	<b>4.3</b>	50.0

**Table 1:** Best-fit point, confidence intervals, pulls for the SM hypothesis and  $p$ -value for different 1D NP scenarios, including  $b \rightarrow \text{see}$  data but assuming NP only in  $b \rightarrow s\mu\mu$ .

can interfere with a NP signal is sketched in Sec. 3. Finally, in Sec. 4 we present a set of new observables able to test lepton-flavour universality violations (LFUV) [9] and briefly review its properties.

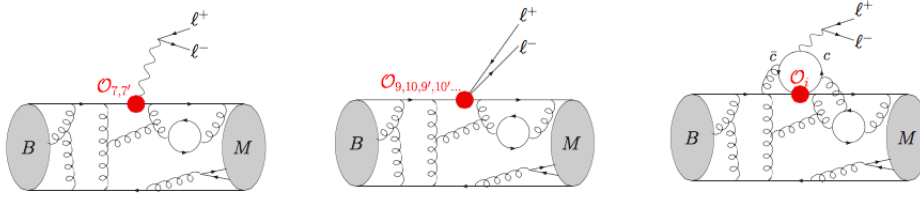
## 2. A short review of the global fits

The observables included in our fit [8] are the branching ratios and angular observables for  $B \rightarrow K^* \mu^+ \mu^-$  and  $B_s \rightarrow \phi \mu^+ \mu^-$ , the branching ratios of the charged and neutral modes  $B \rightarrow K \mu^+ \mu^-$ , the inclusive branching ratios  $B \rightarrow X_s \mu^+ \mu^-$ ,  $B \rightarrow X_s \gamma$  and  $B_s \rightarrow \mu^+ \mu^-$ , as well as the isospin asymmetry  $A_I$  and the time-dependent CP asymmetry  $S_{K^* \gamma}$  of  $B \rightarrow K^* \gamma$ . For the theoretical predictions, lattice form factors from Refs. [10, 11] are used in the low-recoil region, while we resort to light-cone sum rule (LCSR) form factors from Ref. [12], with their correlations assessed from the large-recoil symmetries, in the low- $q^2$  region. The only exception being  $B_s \rightarrow \phi$ , which requires the use of the form factors in Ref. [13].

The hypothesis tested in our analysis is modeled by treating the NP contributions to the Wilson coefficients  $\{C_i^{\text{NP}}\}$  as parameters allowed to vary freely. We estimate the value of these parameters by performing a frequentist fit including experimental and theoretical correlation matrices. In Tab. 1 we present our most updated 1D fit results (only largest pull scenarios shown), where the new  $B_s \rightarrow \phi$  form factors [13] and the new experimental results on  $\mathcal{B}(B^0 \rightarrow K^*(892)^0 \mu^+ \mu^-)$  [14] are included. We report no significant changes in the fit results from the ones in Ref. [8]. In the last two columns we provide information about the goodness-of-fit by displaying the SM-pull, i.e. the number of standard deviations by which the best fit point is preferred over the SM point  $\{C_i^{\text{NP}}\} = 0$ , and the  $p$ -value for each scenario. Therefore, results in Tab. 1 establish the hypothesis of having a contribution to the  $C_9$  coefficient of  $\sim -25\%$  w.r.t. its SM value as the most favoured one [8, 15].

## 3. Some comments about hadronic uncertainties: factorizable power corrections and long distance charm-loops.

Theoretical computations in the framework of semi-leptonic  $B$  decays necessarily require accounting for contributions coming from QCD effects both of perturbative and non-perturbative nature. From the amplitude level perspective, predictions involve tree-level diagrams with insertions of the operators  $\mathcal{O}_{7,9,10}$  (generated at loop level in the SM), as well as one-loop diagrams



**Figure 2:** Illustration of factorisable (first two diagrams) and non-factorisable (third diagram) QCD corrections to exclusive  $B \rightarrow M \ell^+ \ell^-$  matrix elements.

with an insertion of the charged-current operator  $\mathcal{O}_2 = [\bar{s}\gamma^\mu P_L c] [\bar{\ell}\gamma_\mu P_L \ell]$  (generated at tree level in the SM). In contributions of the first type, the leptonic and the hadronic currents factorise, and QCD corrections are restricted to the hadronic  $B \rightarrow M$  current (first two diagrams in Fig. 2). This class of *factorisable QCD corrections* thus forms part of the hadronic form factors parametrizing the  $B \rightarrow M$  transition. On the other hand, contributions of the second type receive *non-factorisable QCD corrections* (third diagram in Fig. 2) that cannot be absorbed into form factors.

*Form factor uncertainties and power corrections:* In order to control uncertainties stemming from factorisable QCD corrections Ref.[16, 17] enclosed in the form factors, one can exploit the large-recoil symmetries of QCD to build observables such that their form factor sensitivity is minimised [2, 3, 18, 19, 20, 21]. Following this line of thought, a set of observables that only exhibit a mild form factor dependence (suppressed by powers of  $\alpha_s$  and  $\Lambda/m_b$ ) was proposed, the so-called optimised observables  $P_i$ . It is essential for the analysis of the observables to control the correlations among the different form factors. For that purpose, our approach relies on the assessment of the correlations by means of the large-recoil symmetry relations. This method guarantees a model independent determination of the correlations from first principles but, as a drawback, the result is only valid up to order  $\Lambda/m_b$  corrections. Factorisable power corrections can be determined using different LCSR computation (Refs. [12, 13]), and the associated error is taken uncorrelated (to be less model dependent). In agreement with the fits is estimated to be  $\mathcal{O}(\Lambda/m_b)$  times the form factor. The method was fully developed in Ref. [17]. The size of the factorisable power corrections has been under intense debate recently due to attempts to explain the LHCb anomalies. This possibility has been fully dismissed by showing the importance of scheme's choice and correlation arguments provided in Ref. [22].

*Impact of long-distance  $c\bar{c}$  loops:* Contributions to the amplitude coming from insertions of the  $\mathcal{O}_2$  effective operator are of non-factorisable type, meaning that cannot be encoded into form factors. These contributions, commonly referred as long-distance charm-loop effects, can mimic a shift in the Wilson coefficient  $C_9$  and thus have been suggested as a solution of the anomaly in  $B \rightarrow K^* \mu^+ \mu^-$  [23]. Unlike a NP contribution, due to the non-local structure of the mentioned corrections, they are expected to show a  $q^2$ -dependence ( $q^2$  stands for the dilepton invariant mass). This contribution always accompanies the perturbative SM part of  $C_9$ , and enter in the structure of the effective Wilson coefficient like  $C_9^{\text{eff}i}(q^2) = C_9^{\text{effSM}}(q^2) + C_9^{\text{NP}} + C_9^{\text{c}\bar{\text{c}}i}(q^2)$  with  $i = \perp, \parallel, 0$ . We have implemented two type of tests to control these contributions [22]. First, a bin-by-bin analysis of the global fit [8] that does not find any indication of a residual  $q^2$ -dependence. And second, we have performed a frequentist fit in [22] parametrizing a possible  $c\bar{c}$  dynamics in a polynomial

expansion and we found results in agreement with KMPW (Ref.[12]) with no hint of any missing large- $q^4$  dependent contribution. Also LHCb analysed the impact of the tails of the resonances on the large-recoil region of  $B^+ \rightarrow K^+ \mu^+ \mu^-$  and found a very tiny effect [24].

#### 4. Observables testing lepton-flavour universality violation

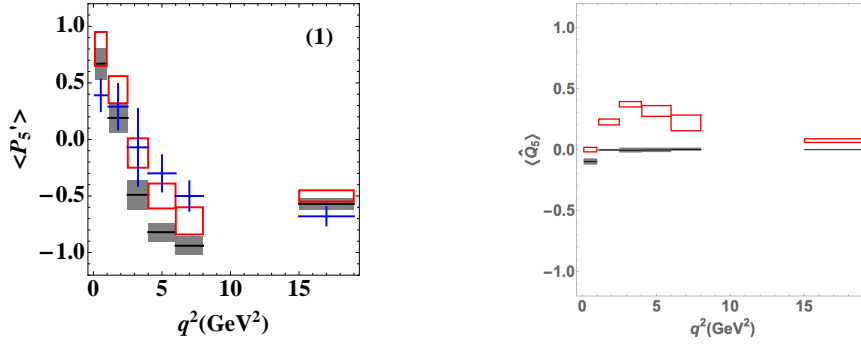
If one focus exclusively to  $B \rightarrow K^* \mu^+ \mu^-$  data, one might be tempted to explain the anomaly in terms of long-distance charm, even despite all the arguments that support the opposite [22]. However, this temptation could just be the result of a excessively narrow point of view. Actually, just by broadening the frame to include  $B \rightarrow K^* e^+ e^-$  data, the hypothesis of long-distance charm effects as the responsible for the observed tensions starts to loose weight, because of  $R_K$ . The deviation observed in this observable cannot be explained by long-distance charm, while it adds coherently with the pattern of deviations observed in  $B \rightarrow K^* \mu^+ \mu^-$ , being possible to explain both tensions by introducing a signal of LFUV that couples only to the muonic channel and not to the electronic one. In other words, both the tensions in  $B \rightarrow K^* \mu^+ \mu^-$  and in  $R_K$  are alleviated by adding a constant contribution to  $C_{9\mu}$  but leaving  $C_{9e}$  SM-like. But  $R_K$  alone, though, is not enough, since it has very little discrimination power among different generic NP scenarios. It is thus necessary to complement  $R_K$  with more observables able to test for LFUV. We proposed in [9] several of them:  $Q_i$ ,  $B_i$  and  $M$ .

##### 4.1 $Q_i$ a new basis of LFUV observables and more

A particularly interesting set of observables with the desired properties can be constructed by comparison of  $P_i^{(\prime)}(B \rightarrow K^* \mu \mu)$  and  $P_i^{(\prime)}(B \rightarrow K^* e e)$  observables. These observables are the so-called  $Q_i = P_i^{(\prime)\mu} - P_i^{(\prime)e}$  [9]. Being defined in terms of optimised observables, the  $Q_i$  inherit their properties and show a reduced sensitivity to hadronic uncertainties. In particular, these observables are protected against long-distance charm-loop contributions in the SM, since the effective operator  $\mathcal{O}_2$  couples identically to muons and electrons. A measurement of  $Q_i$  different from zero would point to NP in an unambiguous way, confirming the violation of LFU observed in  $R_K$ . Obviously, in presence of NP the problem of hadronic uncertainties reemerges, but then we enter in a completely different battle ground, that of a NP Discovery. To illustrate this crucial property of  $Q_i$ , in Fig. 3 we compare  $P_5^{\prime\mu}$  and  $Q_5$  both in the SM and in a NP scenario with  $C_{9\mu}^{\text{NP}} = -1.1$  (taking  $C_{9e}^{\text{NP}} = 0$ ). Notice how tiny are the uncertainties in the SM predictions (grey boxes) for  $Q_5$  compared with  $P_5^{\prime\mu}$ . On the other hand, when we allow for a NP contribution, the theoretical uncertainties in  $Q_5$  grow, as expected, but are more limited than the ones of  $P_5'$ .

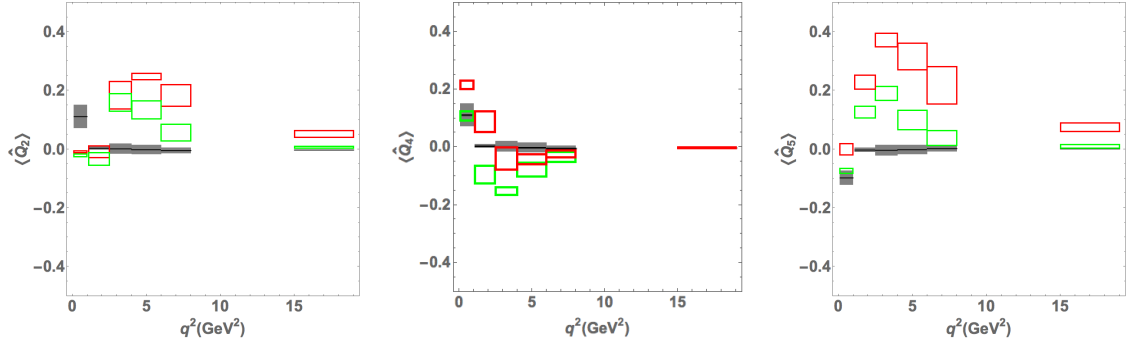
Under the assumption of maximal LFUV between the muonic and electronic modes, the chronic dichotomy of NP or charm is traded by the scenario of a Discovery, where charm-loop uncertainties only enter into the picture when discussing the type of NP. Precisely, for the purpose of distinguishing between potential NP scenarios, the angular analysis of a subset of  $Q_i$  observables ( $i = 1, 2, 4, 5$ ) provides precious information. The observables<sup>2</sup>  $\hat{Q}_1$  and  $\hat{Q}_4$  offer excellent tests for the presence of right-handed currents in  $C_{9\mu}'$  and  $C_{10\mu}'$ , as it can be seen by the very distinctive signature of  $\hat{Q}_1$  and the position of the bins in each end of the large-recoil region of  $\hat{Q}_4$  (see [9]).

<sup>2</sup>The hat notation specifies that  $F_L$  is assumed to be the one measured by LHCb (see [9] for definitions).



**Figure 3:** Predictions for  $P'_5$  and  $Q_5$  in the SM (grey boxes) and in presence of NP (red boxes) in  $C_{9\mu}^{\text{NP}} = -1.1$  (with  $C_{9e}^{\text{NP}} = 0$ ).

Additional tests for distinguishing scenarios with NP contributions only in  $C_{9\mu}$  from scenarios that allow for NP both in  $C_{9\mu}$  and  $C_{10\mu}$  can be found in the last two bins of the large recoil region of  $\hat{Q}_2$  and  $\hat{Q}_5$  and the first two bins of  $\hat{Q}_4$  (see Fig. 4). Belle has been the first experiment to measure the observable  $Q_5$  [25], finding a  $1.2\sigma$  tension w.r.t. the SM prediction in the relevant bin  $[4, 8] \text{ GeV}^2$  (this tension is reduced to  $0.6\sigma$  in the presence of a NP contribution  $C_{9\mu}^{\text{NP}} = -1.1$ ). The low statistical significance of this result makes impossible to draw any conclusion, although it is interesting to notice its connection with the deviation in  $R_K$ .



**Figure 4:** Predictions for  $\hat{Q}_2$ ,  $\hat{Q}_4$  and  $\hat{Q}_5$  in the SM (grey boxes) and in presence of NP (red boxes) in  $C_{9\mu}^{\text{NP}} = -1.1$  and  $C_{9\mu}^{\text{NP}} = -C_{10\mu}^{\text{NP}} = -0.65$  (with  $C_{ie}^{(\prime)\text{NP}} = 0$ ,  $i = 9, 10$ ).

One can also think of exploiting the angular coefficients in electron and muon modes,  $J_i^e$  and  $J_i^\mu$ , in order to build observables sensitive to certain Wilson coefficients, but insensitive to long-distance charm contributions in the SM. Following this idea, in Ref. [9] we proposed  $B_5 = J_5^\mu/J_5^e - 1$ ,  $B_{6s} = J_{6s}^\mu/J_{6s}^e - 1$  and  $M = (B_5 B_{6s})/(B_{6s} - B_5)$ . The observables  $B_5$  and  $B_{6s}$  are form factor independent at all orders (up to corrections related the different lepton masses but suppressed by the dilepton mass) in the SM and share the charm-loop protection properties of the  $Q_i$  observables. In addition, assuming absence of right-handed currents,  $B_5$  and  $B_{6s}$  are proportional to the difference  $(C_{10\mu} - C_{10e})$  which provide them with unique capabilities for testing NP effects in  $C_{10}$ .

Finally, the last LFUV observable proposed in Ref. [9] is the  $M$  observable. This observable has the very singular property of being insensitive to contributions coming from long-distance



charm-loops not only in the SM but also in presence of NP only in  $C_9$ , assuming *transversity-independent charm*<sup>3</sup>. If we consider *transversity-dependent charm* contributions or allow for the presence of NP in  $C_{10}$ , charm effects reemerge in  $M$ . Even though, its strong shielding from hadronic uncertainties makes  $M$  very sensitive to NP at low- $q^2$ .

## 5. Conclusions

Recent experimental results provided by the LHCb and Belle experiments are showing a pattern of tensions with SM predictions in several  $b \rightarrow s \ell^+ \ell^-$  decay modes. Global analyses including all the available data show that scenarios with a large negative  $C_{9\mu}^{\text{NP}}$  are preferred over the SM by typically more than  $4\sigma$ . Explanations for the anomalies in terms of non-factorisable power corrections, are clearly disfavored by the theoretical arguments provided in [22] and also by the appearance of tensions in the LFUV observables  $R_K$  and  $Q_5$ , measured by LHCb and Belle respectively. If confirmed, this tensions would unambiguously point to a NP scenario involving different couplings for muons and electrons. The current situation urges for the measurement of the newly proposed lepton flavour universality tests.

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<sup>3</sup>Charm-loop contributions that do not depend on the helicity of the final state,  $C_9^{c\bar{c}} = C_9^{c\bar{c}\perp} = C_9^{c\bar{c}\parallel} = C_9^{c\bar{c}0}$ .

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